16. Yu. A. Samoilovich, "The possibility of melt crystallization in a self-oscillation regime," Teplofiz. Vys. Temp., 17, No. 5 (1979).
17. V. V. Sobolev and P. M. Trefilov, "Periodic crystallization of a binary alloy," Fiz.-Khim. Obrab. Mater., No. 5 (1984).
18. Yu. A. Samoilovich, S. A. Krulevetskii, V. A. Goryainov, and Z. K. Kabakov, Thermal Processes in Continuous Steel Casting [in Russian], Metallurgiya, Moscow (1982).

ABSORPTION OF SOUND NEAR A SEMIINFINITE RIGID PLANE

V. A. Murga

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1. Research was conducted into the absorption of sound in a viscous heat-conducting compressible fluid (or gas) in the familiar work by Konstantinov [1] for the reflection of a plane sound wave from an infinite rigid plane. The absorption factor in this case, defined as the ratio of the absorbed energy to the incident energy, given small angles of incidence ($\alpha \ll 1$), is equal to

$$d = 4M/(1 + 2M + 2M^2), \tag{1.1}$$

where $M = k_0(\nu/2\omega)^{1/2}/\alpha$; $k_0 = \omega/c$ (ω is the angular frequency of the oscillations and c is the speed of sound); ν is the coefficient of the kinematic viscosity of the fluid (for the sake of brevity, here and below we will assume that the dissipation of the sonic energy is governed exclusively by the viscosity of the medium); moreover, it is assumed that $k_0(\nu/\omega)^{1/2} \ll 1$. Of particular interest is the behavior of the coefficient d in the angles-of-incidence region $\alpha \lesssim k_0(\nu/\omega)^{1/2}$: it changes sharply with respect to the angle and when $\alpha = k_0(\nu/\omega)^{1/2}$ attains a maximum equal to $2(\sqrt{2} - 1)$, and is it not dependent on the properties of the fluid and on the frequency of oscillation (the Konstantinov effect [2]).

Let us note that for a real case of a finite plate in precisely this area of angles of incidence formula (1.1) accurately reflects the process which takes place at such a great distance from the edge of the plate, where the incident wave itself is virtually attenuated owing to absorption in free space. Indeed, we know from the theory of diffraction that a reflected wave near the surface of a plate may be regarded as plane (as was assumed in [1]) at a distance x from the edge of the plate such that the condition $\alpha^2 k_0 x \gg 1$ is satisfied. The coefficient of sound absorption in free space, i.e., $\gamma = 2k_0^3\nu/\omega$ [3], so that if $\alpha \sim k_0(\nu/\omega)^{1/2}$, the indicated condition assumes the form of $\gamma x \gg 1$, which indicates the strong attenuation of the incident wave at a distance x along the plate.

In order to investigate the sound absorption near a finite plate in the case of any small angle of incidence and at such distances from the edge of the plate that the incident wave has not yet been attenuated, it is necessary to drop the assumption that the "reflected" field is a plane wave. This study has been undertaken in the present paper for the case of a semiinfinite plane. It has been demonstrated that the effect of viscosity and the condition of adhesion at the plane leads to a unique "waveguide" effect which consists of the

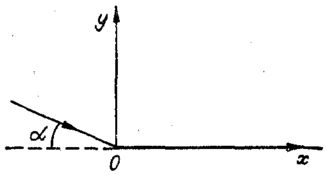

Fig. 1

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fact that a portion of the sonic energy is propagated along the plane, not moving away from it, in the form of a nonuniform wave; this phenomenon has no analog in the acoustics of an ideal fluid.

2. Let a plane monochromatic sound wave impinge on a semiinfinite rigid plane with an incident angle α (the two-dimensional problem). The position of the coordinate axes and the direction of the incident wave (indicated by the arrow) are shown in Fig. 1. The x axis is directed along the plane from the leading edge. The velocity of the fluid particles (with unit amplitude) in the incident wave is given by the expression

$$\mathbf{v}_0 = \frac{\mathbf{k}}{k} e^{ik(x\cos\alpha - y\sin\alpha)} \tag{2.1}$$

[$\mathbf{k}$ is the wave vector, $k = k_0 + i\gamma$, the time factor $\exp(-i\omega t)$ has been dropped throughout]. In order to find the "reflected" field, we will resort to the Kirchhoff theory [4], according to which the field of the velocity vector $\mathbf{v}$ for the particles of a viscous compressible fluid executing small oscillations is described by the equations

$$\nabla^2\mathbf{v}' + k^2\mathbf{v}' = 0, \quad \nabla \times \mathbf{v}' = 0, \quad \nabla^2\mathbf{v}'' + \frac{i\omega}{\nu}\mathbf{v}'' = 0, \quad \nabla\cdot\mathbf{v}'' = 0, \tag{2.2}$$

where $\mathbf{v} = \mathbf{v}' + \mathbf{v}''$. In all the following we will examine the field in the region $y > 0$. Following the Fourier method we will look for the solution for the longitudinal u' (along the x axis) and the transverse v' components of the vector $\mathbf{v}'$ in the form

$$u' = \int_{-\infty}^{\infty} A_\varkappa e^{i(\varkappa x + \mu y)}\, d\varkappa, \quad v' = \int_{-\infty}^{\infty} A_\varkappa \mu e^{i(\varkappa x + \mu y)}\, d\varkappa/\varkappa, \tag{2.3}$$

where A_κ is an arbitrary function of κ; $\mu = (k^2 - \kappa^2)^{1/2}$; the imaginary part of μ must be positive, in order for expression (2.3) to have any physical sense as $y \to \infty$. It is obvious that formulas (2.3) satisfy the first pair of equations in (2.2). Analogously, for the longitudinal and transverse components of the vector $\mathbf{v}''$ we have

$$u'' = \int_{-\infty}^{\infty} B_\varkappa e^{i(\varkappa x + \sigma y)}\, d\varkappa, \quad v'' = -\int_{-\infty}^{\infty} \varkappa B_\varkappa e^{i(\varkappa x + \sigma y)}\, d\varkappa/\sigma \tag{2.4}$$

[$\sigma = (i\omega/\nu - \kappa^2)^{1/2}$ (the imaginary part is positive), B_κ is an arbitrary function of κ]. Expressions (2.4) satisfy the second pair of equations in (2.2), and u'' and v'' differ markedly from zero only in the area of a boundary layer having a thickness of $\sim(\nu/\omega)^{1/2}$.

The boundary conditions for $y = 0$ will be formulated as follows. For $x > 0$ the following condition of adhesion must be satisfied:

$$u' + u'' + u_0 = 0, \; v' + v'' + v_0 = 0$$

[u_0 and v_0 are the longitudinal and transverse components of the vector $\mathbf{v}_0$ in (2.1)]. The velocity of the fluid particles on the ray $y = 0$, $x < 0$ is unknown. We might assume in an ideal fluid that the reflected field is not present in this case. The solution will then be valid near the boundaries separating total shadow from the shadow of the reflected wave (as well as under the condition $k_0 x \gg 1$) [5]; in particular, with small angles of incidence the solution is valid for the region in which the condition $y/x \ll 1$ is satisfied. Total validation of the asserted contention is obtained through solution of the problem of the diffraction of an electromagnetic wave at a conducting plane (Sommerfeld [6]).

In the case of a viscous fluid the assumption to the effect that there is no reflected wave on this ray, strictly speaking, is incorrect; however, it might be assumed to be valid in first approximation, and this follows out of the following considerations. From the standpoint of physical considerations it is clear that the effect of viscosity in the region $x < 0$ makes itself apparent only in the immediate vicinity of the leading edge, in a "viscous" region of dimensions $\sim(\nu/\omega)^{1/2}$; in the remaining area the field, in first approximation, is the same as in the case of an ideal fluid. This is one of the fundamental positions on which boundary-layer theory is based. Therefore, if we adopt $u' + u'' = 0$, $v' + v'' = 0$ as the boundary conditions on the ray, then in view of the above-stated assertions they are invalid for the viscous region, where u'' and v'' are noticeably different from zero. Since the dimensions of the viscous regions are significantly small in comparison to the other characteristic dimensions of the problem, the length of the sound wave [the ratio of these quantities to the parameter $\varepsilon = k_0(\nu/\omega)^{1/2} \ll 1$], it is to be expected, in first approximation, that the viscous region on the ray will have virtually no effect on the solution of the problem, this influence being noticeable only in the subsequent approximations (which are not dealt with in this study). The foregoing follows out of the general positions of the theory of perturbations. Thus, the first approximation must represent the first term in the expansion of the exact solution of the problem in series over powers of the small parameter ε. The subsequent term of the expansion is proportional to ε in the first degree.

The cited validation of the assumed boundary conditions on the ray $y = 0$, $x < 0$, cannot, of course, be regarded as rigorous; however, from the physical standpoint they are rather obvious.

Using (2.3) and (2.4), as well as the expansion of the following functions into the Fourier integral, and namely

$$f(x) = \begin{Bmatrix} v_0 = \frac{\mathbf{k}}{k} e^{ikx\cos\alpha}, & x > 0 \\ 0, & x < 0 \end{Bmatrix} = \frac{\mathbf{k}}{k}\frac{i}{2\pi}\int_{-\infty}^{\infty} e^{i\varkappa x}\frac{d\varkappa}{k\cos\alpha - \varkappa},$$

by means of the boundary conditions for $y = 0$ for A_κ and B_κ we have

$$A_\varkappa + B_\varkappa = -\frac{i\cos\alpha}{2\pi(k\cos\alpha - \varkappa)},$$

$$\mu A_\varkappa/\varkappa - \varkappa B_\varkappa/\sigma = \frac{i\sin\alpha}{2\pi(k\cos\alpha - \varkappa)}.$$

From this, finding A_κ and using (2.3), after simplifications associated with the conditions $\alpha \ll 1$ and $k_0(\nu/\omega)^{1/2} \ll 1$, we find that

$$u' = \frac{i}{2\pi}\int_{-\infty}^{\infty}\frac{(\varkappa\alpha - \varkappa^2\sqrt{\nu/i\omega})\, e^{i(\varkappa x + \mu y)}}{(k\cos\alpha - \varkappa)(\mu + \varkappa^2\sqrt{\nu/i\omega})}\,d\varkappa \qquad (2.5)$$

($\cos\alpha = 1 - \alpha^2/2$). Expression (2.5) describes the reflected field (the longitudinal component of the fluid-particle velocity vector) in the potential region, i.e., outside of the boundary layer (since here $u'' = 0$). We will drop the primes in the following.

The imaginary part of $\mu = (k^2 - \kappa^2)^{1/2}$ is positive for all (real) κ, provided we select that branch of the two-valued function of $\mu(\kappa)$ (in the plane of the complex variable κ) which exhibits a positive imaginary part on the segment of the real axis $-\omega/c \le \kappa \le \omega/c$, and if we make sections at the branching points $\kappa = k$ and $\kappa = -k$ parallel to the imaginary axis, both above and below. The selected μ branch corresponds to the upper sheet of a Riemann surface.

In order to calculate the integral in (2.5) we will deform the original contour of integration in the plane of the complex variable κ, drawing it upward to infinity. It can be demonstrated that the integral vanishes along the infinitely elongated portion of the contour, and it remains only to perform the integration along the edges of the section (Fig. 2). Moreover, with such deformation the contour intersects the pole of the function in

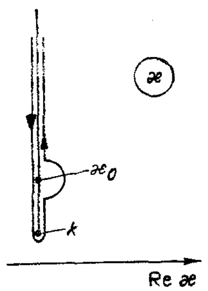

Fig. 2

(2.5) beneath the integral sign at the point $\kappa = k\cos\alpha$, the contribution to the solution from the remainder at this pole represented by the following:

$$u_1 = \frac{\sqrt{i}\,b - 1}{\sqrt{i}\,b + 1} e^{ik(x\cos\alpha + y\alpha)} \tag{2.6}$$

$[b = \alpha(\omega/k_0{}^2\nu)^{1/2}]$. The derived expression describes a plane uniform wave reflected from the plate in accordance with the laws of geometric acoustics (see [1]).

The function in (2.5) beneath the integral sign has yet another pole (in the upper half plane) at the point

$$\varkappa_0 = k + ik_0^3\nu/2\omega \tag{2.7}$$

with an accuracy to the drop small quantities of higher order (relative to the parameter $k_0{}^2\nu/\omega$). It is significant that the second term in the right-hand side of (2.7) is a quantity of the same order of magnitude as the coefficient of absorption γ, representing the imaginary portion of the wave number k. This pole is situated on the section line (Fig. 2). In view of the choice of the branch for the function $\mu(\kappa)$ the denominator in the expression (2.5) beneath the integral sign vanishes on approach to the pole on the right-hand side of the section; thus, the pole is positioned in the right-hand upper (or left lower, which is the same) edge of the section. The integration contour (over the upper sheet of the Riemann surface) is illustrated in Fig. 2. The small semicircle makes it possible to bypass the singularity on the right-hand side of the section edge. Integration over the small semicircle (as its radius tends to zero) yields

$$u_2 = \frac{1}{1 + \sqrt{i}\,b} e^{i(\varkappa_0 x + \mu_0 y)}$$
$$(\mu_0 = (k^2 - \varkappa_0^2)^{1/2} = k_0^2(\nu/2\omega)^{1/2}(i - 1)). \tag{2.8}$$

Expression (2.8) describes a plane nonuniform wave propagated along the plate, with an amplitude dependent on the angle of the incident wave, with an absorption factor equal to $\mathrm{Im}\,\kappa_0 = \gamma + k_0{}^3\nu/2\omega = 7k_0{}^3\nu/6\omega$ [see (2.7)]. If $\alpha \gg k_0(\nu/\omega)^{1/2}$, then $b \gg 1$ and the amplitude of the wave (2.8) is small; if $\alpha \lesssim k_0(\nu/\omega)^{1/2}$, then the amplitude is equal to unity in terms of order of magnitude (when $\alpha = 0$ the amplitude in terms of accuracy is equal to unity). In the case of an ideal fluid ($\nu = 0$, $b = \infty$), $u_2 = 0$.

The existence of the wave in (2.8) is associated with the influence exerted by the boundary layer on the potential region. This effect, it turns out, leads to a situation in which a region of width $\sim(\omega/\nu)^{1/2}/k_0{}^2$ adjoining the plane exhibits, in some measure, the property of a waveguide. Namely, if the source of the sound is located in this region, then a portion of the sound energy emitted by that source in the incident angle range $0 \le \alpha \lesssim k_0(\nu/\omega)^{1/2}$ is propagated along the plane, not moving away from the plane, in the form of the wave in (2.8). If $\alpha \gg k_0(\nu/\omega)^{1/2}$, then the incident wave will not enter this interval of angles and the wave in (2.8) is formed as a consequence of the energy emitted by the leading edge of the plate (within this interval of angles). With $\alpha \lesssim k_0(\nu/\omega)^{1/2}$ it is the incident wave itself that participates in the formation of the wave, which results in a pronounced increase in the amplitude of the wave in (2.8).

For purposes of calculating the integral in (2.5) over the remaining portion of the deformed contour, in addition to the conditions $\alpha \ll 1$ and $k_0(\nu/\omega)^{1/2} \ll 1$ we will use the conditions $k_0x \gg 1$ and $k_0y \ll (k_0x)^{1/2}$. The latter makes it possible to replace $\exp(i\mu y)$ by the expression $1 + i\mu y$; a slight addition to unity is retained to refine the dependence of the solution on y. After transformation the integral over both edges of the section is written in the form

$$u = \frac{2i}{\pi}\left(1 - \sqrt{i}\,b\right) e^{i(kx + \mu_0 y)} \int_0^\infty \frac{\xi^2 e^{-a^2\xi^2}\,d\xi}{(\xi^2 - ib^2)(1 - \xi^2)}, \tag{2.9}$$

where $a^2 = k_0{}^3\nu x/2\omega$, $\xi^2 = 2s\omega/k_0{}^3\nu$, $\kappa = k + is$ (s is real). The integral in (2.9) is understood in the sense of its principal significance, since the integration segment within the infinitely small semicircle has been eliminated, where that semicircle encompasses the pole (at the point $\xi = 1$). Calculation of this integral leads to two terms; the first term

$$u_3 = -\frac{2b}{\sqrt{\pi}(1+\sqrt{i}\,b)} e^{i(kx-w^2+\mu_0 y)} \int_w^\infty e^{it^2}\,dt \tag{2.10}$$

$[w = ab = \alpha(k_0x/2)^{1/2}]$ represents diffraction distortion. In the case of an ideal fluid ($\nu = \mu_0 = 0$, $b = \infty$) Eq. (2.10) changes into the expression

$$u_3 = -\frac{2}{\sqrt{\pi i}} e^{i(kx-w^2)} \int_w^\infty e^{it^2}\,dt, \tag{2.11}$$

which coincides with the above-cited Sommerfeld solution [6] (for $\alpha \ll 1$) when the magnetic-field strength vector is polarized parallel to the leading edge of the plate; it is precisely in this case that we achieve total analogy between diffraction of the electromagnetic wave at the conducting plane and the sound wave (more exactly, the longitudinal components of fluid-particle velocity) at the rigid plane in an ideal fluid. If $\alpha \gg k_0(\nu/\omega)^{1/2}$ ($b \gg 1$), then (2.10) differs weakly from (2.11); however, when $\alpha \lesssim k_0(\nu/\omega)^{1/2}$ the difference becomes radical.

The second term has the form

$$u_4 = \frac{2i}{\sqrt{\pi}(1+\sqrt{ib})} e^{i(\varkappa_0 x+\mu_0 y)} \int_0^a e^{t^2}\,dt, \tag{2.12}$$

and in the case of an ideal fluid it vanishes. At distances, not overly large, from the edge, when $a^2 = k_0{}^3\nu x/2\omega \ll 1$ (here, of course, $k_0x \gg 1$), u_4 is small; when $a \sim 1$, then $u_4 \sim 1$, and in this event the attenuation of the incident wave makes itself felt, owing to absorption in space. When $a \gg 1$ (the incident wave is virtually attenuated)

$$u_4 \approx \frac{i}{\sqrt{\pi}\,a(1+\sqrt{i}\,b)} e^{i(kx+\mu_0 y)}.$$

In the latter case expressions (2.8), (2.10), and (2.12) are small in comparison to (2.6) and, thus, the latter becomes suitable for all angles of incidence, including the case in which $\alpha = 0$.

3. We will calculate the magnitude of the sound energy absorbed in the boundary layer by means of the theorem related to energy dissipation in fluids [3], according to which the energy dissipated in the layer (per unit time and unit area of the plate)

$$E = \eta \int_0^\delta \left(\frac{\partial u}{\partial y}\right)^2 dy, \tag{3.1}$$

where $\eta = \nu\rho$; ρ is the density of the fluid; u is the longitudinal component (real) of the velocity in the boundary layer; δ is the thickness of the boundary layer. Since the complex amplitude in the boundary layer is expressed as $u = u_e(1 - \exp[-(i\omega/\nu)^{1/2}y])$ (u_e is the complex longitudinal velocity at the external edge of the boundary layer), then Eq. (3.1), averaged over time, will yield

$$\overline{E} = \frac{1}{2\sqrt{2}} |u_e|^2 \rho (\omega\nu)^{1/2}. \tag{3.2}$$

The quantity u_e is found to be the sum of the terms in (2.6), (2.8), (2.10), and (2.12), and also of u_0 [the incident wave in (2.1)] for $y = 0$. If $a \ll 1$ (the case of greatest interest from the standpoint of application, when attenuation in the incident wave can be neglected), in which case

$$u_e = 2e^{ik_0x}\left(1 - \frac{1}{\sqrt{\pi i}} \int_w^\infty e^{it^2}\,dt\right). \tag{3.3}$$

It is remarkable that although the above-enumerated terms change on the basis of the angle $\alpha \lesssim k_0(\nu/\omega)^{1/2}$ in the region on an individual basis (with the exception of the incident wave) as a consequence of their dependence on viscosity, the resulting velocity (3.3) is independent of α (for $\alpha \ll 1$) and of the viscosity, i.e., u_e behaves precisely as an ideal fluid, which is quite natural, although not obvious from the start. However, this is not the result which we find when the values of the parameter a are not overly small.

The coefficient d in the boundary layer is obtained through division of $\bar{E}$ in (3.2) by the time-averaged magnitude of the energy flux in the incident wave, referred to a unit of plate area, i.e., by $\rho c|\mathbf{v}_0|^2\alpha/2$; thus, we have

$$d = \frac{|u_e|^2}{\alpha\sqrt{2}}\left(\frac{\nu\omega}{c^2}\right)^{1/2}, \tag{3.4}$$

since $|\mathbf{v}_0| = 1$. In particular, on simultaneous satisfaction of the conditions $a \ll 1$, $\alpha \gg k_0(\nu/\omega)^{1/2}$, $w = ab \gg 1$ it follows from (3.3) that $|u_e| = 2$, and then $d = 2\sqrt{2}(\nu\omega/c^2)^{1/2}/\alpha = 4M$, which coincides with (1.1), since here $M \ll 1$. In the other particular case, when $a \ll 1$ and $\alpha \lesssim k_0(\nu/\omega)^{1/2}$, $w \ll 1$, and from (3.3) it turns out that $|u_e| = 1$, so that

$$d = (\nu\omega/c^2)^{1/2}/\alpha\sqrt{2} = M. \tag{3.5}$$

This expression is valid for all α (in the indicated region of angles), including the case in which $\alpha = 0$. The infinite value of d in (3.5) for $\alpha = 0$ does not indicate "infinitely large" absorption, but is a consequence of the adopted definition of d; the "dimensional" absorption from (3.2) is totally independent of α. Thus, the result obtained in (3.5), as must be the case, differs from the results given in (1.1), which in view of the above is inapplicable to the region of small angles of incidence ($\alpha \lesssim k_0\sqrt{\nu/\omega}$) for such distances x from the edge of the plate at which the incident wave has not yet become attenuated owing to absorption in space ($k_0 x \gg 1$, but $k_0^3\nu x/\omega \ll 1$).

LITERATURE CITED

1. B. P. Konstantinov, "The absorption of sound waves on reflections from a solid boundary," Zh. Tekh. Fiz., 9, No. 3 (1939).
2. V. A. Dulov, "The Brewster effect in acoustics and the Konstantinov effect," Akust. Zh., No. 5 (1980).
3. L. D. Landau and E. M. Lifshits, The Mechanics of Continuous Media [in Russian], Gostekhizdat, Moscow (1954).
4. J. W. Strutt (Lord Rayleigh), The Theory of Sound [Russian translation], GITTL, Moscow, Vol. 2 (1955).
5. L. D. Landau and E. M. Lifshits, Field Theory [in Russian], Nauka, Moscow (1967).
6. L. D. Landau and E. M. Lifshits, The Electrodynamics of Continuous Media [in Russian], GIFML, Moscow (1959).